Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **October/November 2024**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Solve the following simultaneous equations.

$$\frac{y}{x} = \frac{3}{2}$$

$$\frac{y^4}{x^5} = \frac{27}{16}$$ [3]

2 Variables x and y are related by the equation $y = x\sqrt{1+2x}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) It is given that when $y = 12$, $x = 4$. Find the approximate change in x when y increases from 12 by the small amount 0.06. [3]

(c) Find the x-coordinate of the stationary point on the curve $y = x\sqrt{1+2x}$. [2]

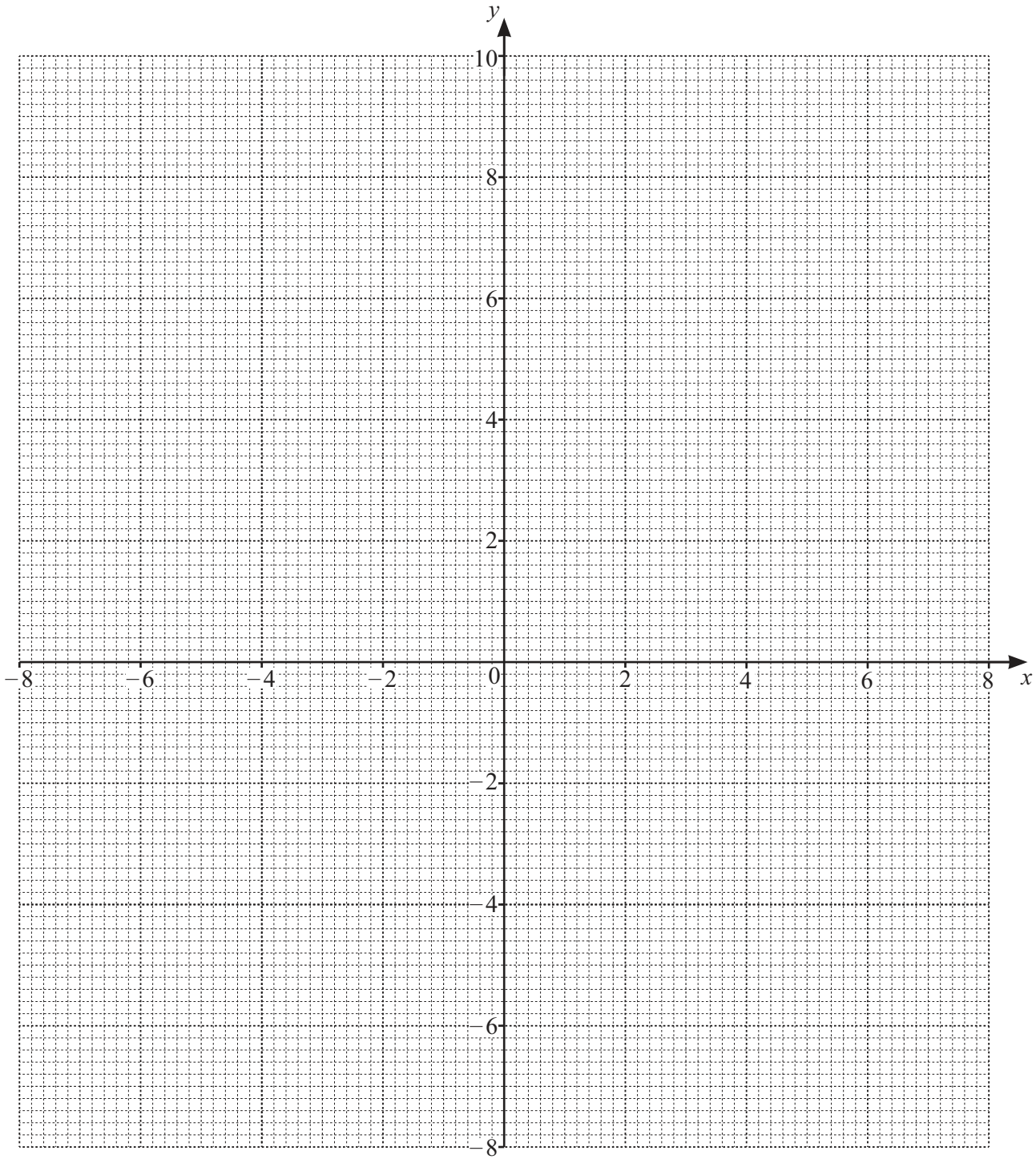
3 DO NOT USE A CALCULATOR IN THIS QUESTION.

The polynomial p is defined by $\mathrm{p}(x) = ax^3 - 3x^2 - 3x + b$, where a and b are constants.

(a) Given that $x = 2$ and $x = -1$ are roots of the equation $\mathrm{p}(x) = 0$, find a and b. [3]

(b) Solve the equation $\mathrm{p}(x) = 0$. [2]

4 Use a graphical method to solve the inequality $|2x-8|>4$. [5]



5 Solve the following equations.

(a) $\log_2 x^2 + \log_{16} x = 18$ [4]

(b) $e^{2x+1} - 10e^{-2x-1} = 3$ [4]

6 DO NOT USE A CALCULATOR IN THIS QUESTION.

Write $(5-\sqrt{3})(\sqrt{6}+\sqrt{2})^{-2}$ in the form $a+b\sqrt{3}$, where a and b are constants. [5]

7 A class of 10 students includes Abby and Ben.

(a) A group of 5 students is to be selected from the class. Find the number of possible groups in the following cases.

(i) There are no restrictions. [1]

(ii) The group includes both Abby and Ben. [2]

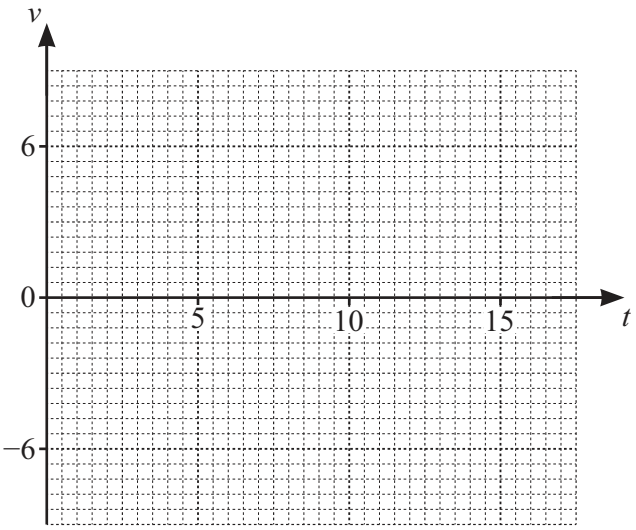
(iii) The group includes either Abby or Ben, but not both. [2]

(b) All 10 students are arranged in a line. How many arrangements are possible if there are exactly three students between Abby and Ben? [3]

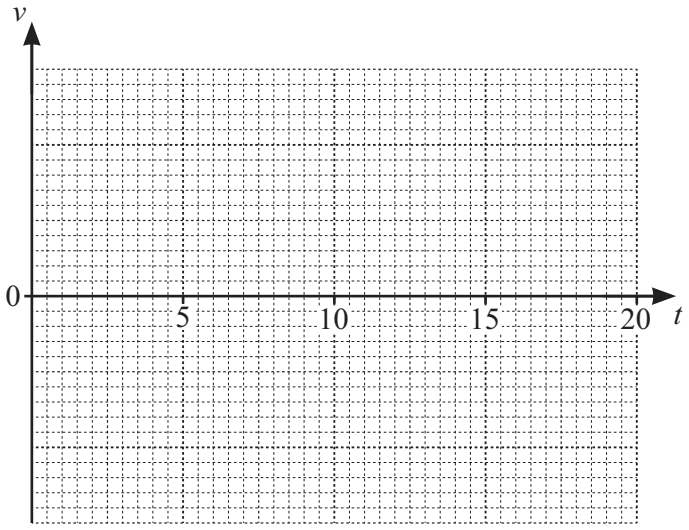
8 Solve the equation $\cot^2 2\theta + 3\operatorname{cosec} 2\theta = 9$ for $-90° \leqslant \theta \leqslant 90°$. [6]

9 In this question time is measured in seconds.

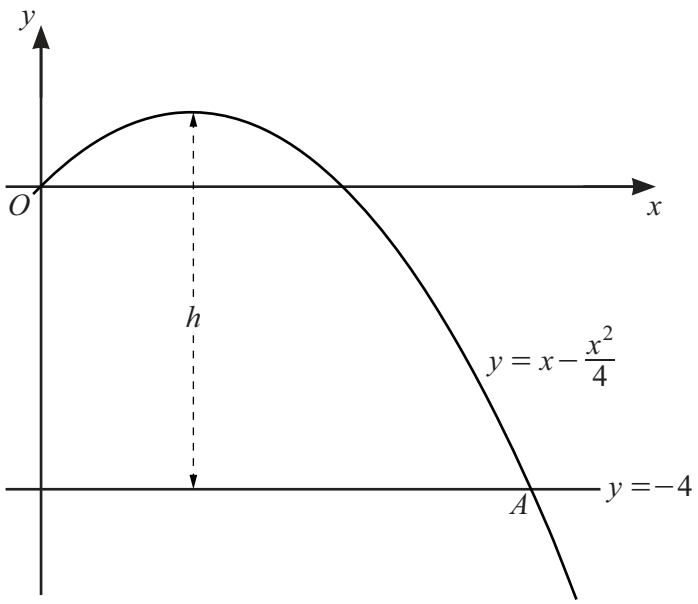
(a) A particle is moving in a straight line with constant velocity of $6\,\text{ms}^{-1}$. At time $t = 0$, it passes a fixed point A. At time $t = 5$ it suddenly changes direction and moves with a different constant velocity along the same straight line. It passes the point A again at time $t = 15$. Sketch the velocity–time graph for the motion. [3]



(b) Another particle is moving in a straight line with constant acceleration. At time $t = 0$ it passes a fixed point B with velocity $-8\,\text{ms}^{-1}$. It passes the point B again at time $t = 20$. Sketch the velocity–time graph for the motion. [3]



10 The diagram shows part of the curve $y = x - \frac{x^2}{4}$ and the line $y = -4$. The curve and the line intersect at the point A.



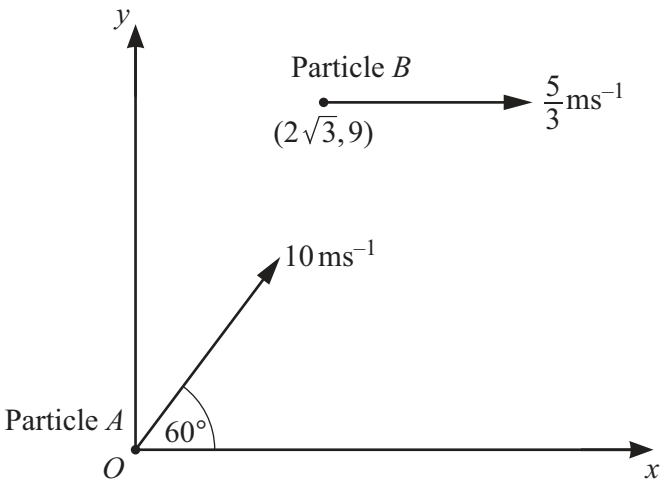
(a) The maximum point on the curve is at a perpendicular distance h from the line $y = -4$. Find the value of h. [4]

(b) Find the exact x-coordinate of A. [3]

(c) Find the acute angle between the tangent to the curve at A and the line $y = -4$. [4]

11 In this question **i** is a unit vector in the positive x-direction and **j** is a unit vector in the positive y-direction. Time is in seconds and distances are in metres.

The diagram shows the initial positions and velocities of two particles, A and B, that move in the x-y plane.



Particle A starts from the origin O at time $t = 0$. It moves with constant speed $10\,\text{ms}^{-1}$ in the direction 60° above the x-axis.

(a) Find the exact values of the components of the velocity of particle A in the x-direction and the y-direction. [2]

(b) Find, in terms of t, the position vector of particle A at time t. [1]

Particle B starts from the point $(2\sqrt{3}, 9)$ at time $t = 0$. It moves with constant speed $\frac{5}{3}\text{ms}^{-1}$ parallel to the positive x-axis.

(c) Find, in terms of t, the position vector of particle B at time t. [2]

(d) Hence show that the particles collide. [4]

Question 12 is printed on the next page.

12 A metal tank is in the shape of a cuboid with a square base of side x m and an open top. The tank has a volume of $5\,m^3$. Given that x can vary, and that the area of the metal used to make the tank is a minimum, find the dimensions of the tank. [6]